ANALYTIC EVALUATION OF THE WEIGHTING FUNCTIONS FOR REMOTE SENSING OF BLACKBODY PLANETARY ATMOSPHERES: THE CASE OF LIMB VIEWING GEOMETRY

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In a recent publication (Ustinov, 2002), we proposed an analytic approach to evaluation of radiative and geophysical weighting functions for remote sensing of a blackbody planetary atmosphere, based on general linearization approach applied to the case of nadir viewing geometry. In this presentation, the general linearization approach is applied to the limb viewing geometry. The expressions, similar to those obtained in (Ustinov, 2002), are obtained for weighting functions with respect to the distance along the line of sight. Further on, these expressions are converted to the expressions for weighting functions with respect to the vertical coordinate in the atmosphere. Finally, the numerical representation of weighting functions in the form of matrices of partial derivatives of grid limb radiances with respect to the grid values of atmospheric parameters is used for a convolution with the finite field of view of the instrument.

Notations

$B(z) = B(T(z))$	Atmospheric Planck function
$B_s = B(T_s)$	Surface Planck function
$f(z)$	Mixing ratio of an atmospheric constituent
H_g	Atmospheric scale height
$I(z, \mu)$	Intensity of radiation
p	Atmospheric pressure; just a parameter
r	Radial distance
r_p	Radial distance to pericenter of line of sight
s	Distance along the line of sight
$T(z)$	Atmospheric temperature
T_s	Surface temperature
$X(z)$	Atmospheric geophysical parameters
$z = \int H_g \ln p$	Vertical coordinate
$\kappa(z)$	Atmospheric absorption coefficient
μ	Zenith angle of the line of sight
τ	Optical depth or distance along line of sight

1. Analytic computation of radiances for non-scattering atmospheres in thermal spectral region

The RT equation: a Beer-Lamberth law:

$$-\frac{dI}{d\tau} + I(\tau) = B(\tau)$$
$$I(\tau_0) = B_0$$

Solution at TOA ($\tau = 0$):

$$R = I(0) = B_0 \exp(-\tau_0) + \int_{\tau_0}^{0} B(\tau)\exp(-\tau)d\tau$$

Using the transmittance function $t(\tau) = \exp(-\tau)$:

$$R = B_0 t(\tau_0) + \int_0^{\tau_0} B(\tau)t(\tau)d\tau =$$

$$= B_0 t(\tau_0) + \int_{\tau_0}^{0} B(\tau)dt(\tau) = B(0) + \int_0^{\tau_0} t(\tau)dB(\tau)$$

The radiance R is an operator, $B(\tau)$ and $t(\tau)$ being its argument functions; in addition, R is a function of two scalar arguments B_s and τ_0.

Corresponding variations:

$$\delta_B R = \int_0^{\tau_0} \delta B(\tau)dt(\tau), \quad \delta_t R = \int_0^{\tau_0} \delta t(\tau)dB(\tau)$$

$$\delta_{B_0} R = t(\tau_0)\cdot \delta B_0, \quad \delta_{t_0} R = B_0 \cdot \delta t(\tau_0)$$

Two remarks at this point:

- All geophysical atmospheric and surface parameters are encapsulated in radiative atmospheric and surface parameters: $B(\tau)$, $t(\tau)$, and B_0, t_0

- As of yet, we didn't specify any geometry of observations.

2. Weighting functions as kernels of linearized operators; Radiative and geophysical weighting functions

The reason we need weighting functions: Need to solve the problems like this one:

$$\int K^{(X)}(\varsigma, p)\Delta X(\zeta)d\zeta = \Delta R(p)$$

To obtain the weighting function with respect to *any* parameter, we need to linearize the radiance operator with respect to this parameter.

We don't need to linearize wrt *all* geophysical parameters of interest. We need to linearize wrt *only* the radiative parameters entering the RT equation.

Then we add partials of radiative parameters wrt geophysical parameters.

For non-scattering atmospheres in thermal spectral region, there are only *two* radiative parameters: describing the source radiation and atmospheric opacity:

$$B(z), \text{ and } \kappa(z)$$

The Planck function B depends only on atmospheric temperature T; the atmospheric opacity depends on the rest of atmospheric parameters X, and may depend on T too. Thus we have:

Temperature weighting function:

$$K^{(T)}(\zeta, p) = K^{(B)}(\zeta, p)\cdot\frac{\partial B}{\partial T} + K^{(\kappa)}(\zeta, p)\cdot\frac{\partial\kappa}{\partial T}$$

Weighing functions for the rest of atmospheric parameters

$$K^{(X)}(\zeta, p) = K^{(\kappa)}(\zeta, p)\cdot\frac{\partial\kappa}{\partial X}$$

(So far, we didn't do any RT computations, and didn't specify the viewing geometry.)

3. Radiative weighting functions for an arbitrary viewing geometry (still nothing specific)

Planck weighting function: We have to linearize the expression

$$R(p) = t(s_0, p)B_0 + \int_0^{s_0} B(s)dt(s, p)$$

with respect to the Planck function B. We have:

$$\delta_B R(p) = \int_0^{s_0}\delta B(s)dt(s, p) = \int_0^{s_0}\delta B(s)\frac{\partial t(s, p)}{\partial s}ds = \int_0^{s_0}\frac{\partial t(s, p)}{\partial s}\frac{ds}{dz}\delta B(z)dz$$

whence

$$K^{(B)}(z, p) = \frac{\partial t(s, p)}{\partial s}\frac{ds}{dz} = \kappa(s)t(s, p)\frac{ds}{dz}$$

Absorption coefficient weighting function: We have to linearize the following chain of expressions with respect to the absorption coefficient κ:

$$R(p) = B(0) + \int_0^{s_0} t(s, p)dB(s)$$

$$t(s, p) = \exp\{-\tau[s(p)]\}$$

$$\tau[s(p)] = \int_0^s \kappa(s')ds(p)$$

After some algebra we have:

$$\delta^{(\kappa)}R(p) = \int_0^{z_0}\left[\int_z^{z_0} t(s,p)dB(s)\right]\delta\kappa(z)dz$$

whence

$$K^{(\kappa)}(z,p) = \int_z^{z_0} t(s,p)dB(s)$$

So, we did some RT computations applicable to any viewing geometry.

Now we consider the specific viewing geometries. We will review the results obtained for the nadir viewing geometry and compare them with those relevant to the limb viewing geometry.

4. Radiative weighting functions for the nadir viewing geometry

Here we have:

$$\frac{ds}{dz} = \frac{1}{\mu}$$

Here μ, the cosine of the zenith angle of LOS also serves as a parameter specifying the viewing conditions. We obtain immediately:

$$K^{(B)}(z,\mu) = \frac{1}{\mu}\kappa(z)t(z,\mu)$$

$$K^{(\kappa)}(z,\mu) = \frac{1}{\mu}\left[r(z,\mu) - t(z,\mu)B(z)\right]$$

If this appears too simple, we can lift the bar. Let's assume that the underlying surface's brightness is not only due to its proper thermal radiance, but also due to reflected downwelling atmospheric radiation:

$$B_0 = \varepsilon B_s + (1-\varepsilon)R_{\downarrow}$$

After some of algebra, we can obtain:

$$K^{(B)}(z,\mu) = \kappa(z)\left[\frac{1}{\mu}t(z,\mu) + t(z_0,\mu)t_{\downarrow}(z)\right]$$

$$K^{(\kappa)}(z,\mu) = \left\{\frac{1}{\mu}\left[r(z,\mu) - t(z,\mu)B(z)\right] + t(z_0)\left[r_{\downarrow}(z) - t_{\downarrow}(z)B(z)\right]\right\}$$

Here $t_{\downarrow}(z)$ and $r_{\downarrow}(z)$ are "diffuse" downwelling transmittances and radiances, computable in the same way as upwelling transmittances and radiances but integrated over the lower hemisphere. The reflected downwelling radiation is usually ignored in computations of weighting functions for nadir-viewing geometry.

5. Radiative weighting functions for the limb viewing geometry

Here, the parameter specifying the LOS is the radial distance of its pericenter, r_p. Also, $ds/dz \neq$ const. Switching to the radial distance, $r = r_0 + z$, we have:

$$s = \pm\sqrt{r^2 - r_p^2}$$

Transmittances along the LOS:

$$t(s, r_p) = \exp\left[-\int_{-\infty}^{s} \kappa(s')ds' \right]$$

$$t_0(r_p) = t(0, r_p)$$

$$t_\downarrow(s, r_p) = t_0(r_p) / t(s, r_p)$$

After some of algebra, we can obtain

$$K^{(B)}(r, r_p) = \frac{ds}{dr}\kappa(z)\left[t(s, r_p) + t_0(r_p)t_\downarrow(s, r_p)\right]$$

$$K^{(\kappa)}(r, r_p) = \frac{ds}{dr}\left\{\left[r(s, r_p) - t(s, r_p)B(z)\right] + t_0(r_p)\left[r_\downarrow(s, r_p) - t_\downarrow(s, r_p)B(z)\right]\right\}$$

The conversion factor ds / dr has a singularity at $s = 0$:

$$\frac{ds}{dr} \underset{s\to 0}{\longrightarrow} \infty$$

This complication is worked around assuming that the model atmosphere can be adequately represented on a discrete altitude grid.

Assuming a linear (or quadrature, or spline) interpolation, one can obtain a working finite-dimensional representation.

After some algebra, we obtain:

$$\Delta R_{j_p} = \int_{r_0}^{\infty} K(r, r_p)\Delta X(r)dr \approx \sum_{j=j_p} K_{j_p j}\Delta X_j$$

Finally, the obtained $K_{j_p j}$ matrix can be convolved with the set of limb FOVs of a given instrument in a same fashion, in which the vector of radiances R_{j_p} computed for the set of lines-of-sight with pericenter indices j_p and pericenter radii r_p. More specifically, we interpolate them over intermediate values of pericenter radii, convolve the interpolation functions (linear, quadratic, cube) with the set of limb FOVs of the instrument and, after some algebra, obtain:

$$\Delta R_i = \sum_j K_{ij}^{(FOV)} \Delta X_j$$

With the Jacobian matrix $K_{ij}^{(FOV)}$ computed, we are now ready to solve the resulting inverse problem

$$\mathbf{Kx} = \mathbf{y}$$

using any applicable inversion method(s).

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Reference:

Ustinov E.A. (2002) Analytic evaluation of the weighting functions for remote sensing of blackbody planetary atmospheres: A general linearization approach. JQSRT, v.73, p.29.